ISSN 2413-5577, Ecological Safety of Coastal and Shelf Zones of Sea, 2025, No. 2, pp. 80–98.

Original paper

Resonance Properties of Water Areas Based on Mathematical Modelling Results: A Case of Sevastopol Bays (the Black Sea)

Yu. V. Manilyuk *, A. Yu. Belokon, A. V. Bagaev, Yu. Yu. Yurovsky, D. I. Lazorenko

Marine Hydrophysical Institute of RAS, Sevastopol, Russian Federation
* *e-mail: uvmsev@yandex.ru*

Abstract
Within linear approximation of the long-wave theory, the paper uses the hydrodynamic numerical model ADCIRC to study resonance properties of the water areas of the Sevastopol bays: Streletskaya, Kruglaya, Kamyshovaya, and Dvoynaya, which includes Kazachya and Solyonaya bays. The calculations were carried out for the water area of each bay separately based on numerical experiments. At the first stage of modelling, waves in the bays water areas were excited by setting red-noise disturbance at the liquid boundary at the bay entrance. At the second stage, free oscillations were calculated under condition of their free passage at the liquid boundary. The resonance periods of the above bays and spatial distribution of the spectral density of sea-level oscillations in their water areas for individual natural modes were determined using spectral analysis. Most of the resonance periods for the Sevastopol bays were in satisfactory agreement with analytical estimates. The use of realistic bathymetry and shoreline profile data in the modelling allowed obtaining additional resonance periods for all the considered bays, which cannot be obtained with analytical estimates. The spectral composition expansion of the resonance modes due to connection of these bays via their entrances was revealed in Dvoynaya Bay, which includes Kazachya and Solyonaya Bays. The spectral density spatial distribution of the main energy-carrying sea level oscillations in Streletskaya, Kruglaya, Kamyshovaya and Dvoynaya Bays was analysed to show that the spectral density peaked mainly in the bay tops. In Dvoynaya Bay, the spectral density maxima manifested in the eastern or western arms (Kazachya or Solyonaya Bays, respectively) depending on which of the arms their natural periods belonged to. The results obtained here can be used in layout designing of hydraulic structures, development of mariculture, planning of wastewater outlets, etc.

Keywords: seiche oscillations, seiche, Sevastopol bays, ADCIRC model, mathematical modelling

Acknowledgements: The study was funded under grant no. 24-27-20076, https://rscf.ru/project/24-27-20076/ of the Russian Science Foundation and Agreement no. 86 as of 19.06.2024 of the Education and Science Department of the City of Sevastopol.

For citation: Manilyuk, Yu.V., Belokon, A.Yu., Bagaev, A.V., Yurovsky, Yu.Yu. and Lazorenko, D.I., 2025. Resonance Properties of Water Areas Based on Mathematical Modelling Results: A Case of Sevastopol Bays (the Black Sea). *Ecological Safety of Coastal and Shelf Zones of Sea*, (2), pp. 80–98.

Резонансные свойства акваторий севастопольских бухт (Черное море) по результатам математического моделирования

Ю. В. Манилюк*, А. Ю. Белоконь, А. В. Багаев, Ю. Ю. Юровский, Д. И. Лазоренко

Морской гидрофизический институт РАН, Севастополь, Россия

* *e-mail: uvmsev@yandex.ru*

Аннотация

В рамках линейного приближения теории длинных волн на основе гидродинамической численной модели *ADCIRC* исследуются резонансные свойства акваторий севастопольских бухт: Стрелецкой, Круглой, Камышовой, Двойной, включающей в себя б. Казачью и Соленую. Расчеты проведены для акватории каждой бухты отдельно на основе численных экспериментов. На первом этапе моделирования возбуждаются волны в акваториях бухт с помощью задания на жидкой границе, находящейся у входа в бухту, возмущения типа «красный шум». На втором этапе рассчитываются свободные колебания с условием свободного прохождения на жидкой границе. С использованием спектрального анализа установлены резонансные периоды указанных бухт и пространственное распределение спектральной плотности колебаний уровня в их акваториях для отдельных собственных мод. Большинство выделенных резонансных периодов для севастопольских бухт удовлетворительно согласуются с аналитическими оценками. Использование при моделировании данных батиметрии и профиля береговой черты, приближенных к реальным, позволило получить для всех рассмотренных бухт дополнительные резонансные периоды, которые нельзя получить при аналитических оценках. В б. Двойной, включающей в себя б. Казачью и Соленую, выявлено расширение спектрального состава резонансных мод, возникающее из-за связи этих бухт через их входы. Анализ пространственного распределения спектральной плотности основных энергонесущих колебаний уровня в б. Стрелецкой, Круглой, Камышовой, Двойной показал, что максимальные значения спектральной плотности возникают в основном в вершинах бухт. В б. Двойной максимальные значения спектральной плотности проявляются в восточном или западном рукавах (б. Казачья или Соленая соответственно) в зависимости от того, к какому из рукавов относится собственный период. Полученные здесь результаты могут быть использованы при проектировании размещения гидротехнических сооружений, развитии марикультуры, планировании выпусков сточных вод и т. п.

Ключевые слова: сейшевые колебания, сейши, севастопольские бухты, модель *ADCIRC*, математическое моделирование

Благодарности: исследование выполнено за счет гранта Российского научного фонда № 24-27-20076, https://rscf.ru/project/24-27-20076/ и Соглашения с Департаментом образования и науки г. Севастополя № 86 от 19.06.2024 г.

Для цитирования: Резонансные свойства акваторий севастопольских бухт (Черное море) по результатам математического моделирования / Ю. В. Манилюк [и др.] // Экологическая безопасность прибрежной и шельфовой зон моря. 2025. № 2. С. 80–98. EDN SNXXDY.

Introduction

The city of Sevastopol is located mainly within the Heraclean Peninsula and its shoreline is indented with many bays and capes. They form a system consisting of main Sevastopol Bay and several smaller adjacent bays. Seiches, i.e. standing water mass oscillations, are often observed in such water bodies. Unlike seiches in completely enclosed basins, they are excited by the penetration of long waves from the open sea through the liquid boundary, with wave energy mainly being lost through radiation at the bay entrance [1]. In contrast to enclosed water bodies, bays also generate the Helmholtz mode (zero mode). This mode usually dominates all other types of natural oscillations and determines the general character of motions in the water area [2, 3]. The danger of seiches in bays is that they can cause intense periodic currents that threaten coastal infrastructure and ships [4]. It is also known that adjacent bays can interact by exchanging energy across their open boundaries, resulting in the penetration of natural modes of one bay into another [5, 6]. The Sevastopol bays form a system of interconnected oscillators, resulting in an expansion of the seiche mode composition in each bay.

Periods of seiche oscillations are determined by geometrical parameters of the water area, such as depth, shoreline outlines and bathymetry. In the Black Sea, it is known that seiche periods in bays and gulfs range from several minutes to two hours, with periods of 5–10 min occurring in all of them. Level oscillations with periods of 2–3 min can be caused by the transformation of long waves in the coastal zone or by sharp gusts of wind in one direction. Oscillations with periods of 15–20 min can be caused by sharp changes in atmospheric pressure as well as changes in wind direction and speed. Seiches of this type most often appear at the passage of cyclones, especially in the peripheral zone, where wind intensification occurs [7]. At the same time, level oscillations with a period of up to 10 min are the most common. Of the seiches with a longer period on the Crimean coast, the most frequent are those with a period of 30–50 min [7]. Analysis of short-term measurements taken with an ADCP probe during expeditions by Marine Hydrophysical Institute in 2008 and 2014 revealed that current oscillations occurred at the entrance to Sevastopol Bay with a period of ~60 min [8].

In addition to seiches, tidal level oscillations inherent to the Black Sea appear in the Sevastopol bays as a background. In [9], devoted to the study of the Black Sea tides, it is indicated that the main lunar semidiurnal component (12.42 h) and the gravitational lunar-solar diurnal component (23.93 h) are the most intense on the Crimean coast. The intensity values of these tidal modes are close to each other, which agrees well with the data from [10].

At present, seiche oscillations in Sevastopol bays remain poorly studied. Data of *in situ* observations are only available for Sevastopol and Kruglaya bays. Sea level oscillations in Sevastopol Bay were recorded using a mareograph located at the marine hydrometeorological station on Cape Pavlovsky [10]. During the processing of the results of *in situ* observations, the periods corresponding to the semidiurnal and diurnal components of the tide as well as with values of 0.9, 1.25, and 2.5 h were identified. The oscillation with a period of 0.9 h is the Helmholtz mode of Sevastopol Bay. In Kruglaya Bay, data were obtained relatively recently (in 2023), with level measurements using an ultrasonic sensor and resonance periods of local seiches identified on the basis of spectral analysis [11]. Among them, the most intense Helmholtz mode of Kruglaya Bay with a period of 13.7 min was identified.

The establishment of physical regularities of level oscillations in a system of connected bays, such as the Sevastopol bays, is possible through mathematical modelling. Several studies [1, 6, 11, 12] have investigated seiche oscillations in Sevastopol bays using numerical modelling. In [6], the mutual influence of Sevastopol and Karantinnaya bays on each other due to the exchange of oscillation energy through their entrances was investigated. The study showed that the intensity of the natural modes of Sevastopolskaya Bay could exceed that of Karantinnaya Bay when they penetrated into the latter. In [1], the resonance periods of the main Sevastopol bays and the influence of the duration of the initial disturbance on seiche generation in them were studied. In [12], different modes of seiche oscillations in Sevastopol Bay were studied in the context of disturbances with periods of 2.5, 2.9 and 6.2 min, which correspond to the natural modes of the bay with different spatial structures. The periods of the fundamental modes of the Sevastopol (50 min) and Karantinnaya (9.25 min) bays were determined based on the numerical solution of the problem for natural values.

Nevertheless, information on the periods and spatial structure of natural level oscillations in all Sevastopol bays remains insufficient. Therefore, it is necessary to develop a comprehensive understanding of long-wave oscillations in the Sevastopol bays and determine the resonance (natural) period values of the Sevastopol bays system and its individual elements within the high-frequency spectrum (hours – minutes).

The purpose of this study is to analyse the resonance response of sea levels in the Sevastopol bays: Streletskaya, Kruglaya, Kamyshovaya, Kazachya and Solyonaya, in response to disturbances from the open sea, and to determine their natural periods. Investigating the resonance properties of each bay separately made it possible to study the mode composition of seiches in detail. Understanding the resonance properties of each bay in the system will enable more accurate interpretation of modelling and measurement results for the entire system.

Materials and methods of study

A detailed study of seiche oscillations in Sevastopol bays was conducted using bathymetric data from digitised nautical charts. Fig. 1 shows the bathymetry of the Sevastopol coastal zone and its bay system.

Numerical modelling was carried out separately for each of the studied bays: Streletskaya, Kruglaya, Kamyshovaya, Kazachya and Solenaya. Fig. 2 shows the bathymetry of these bays. Sea level oscillations were calculated at points *1–23* (Fig. 2).

Streletskaya Bay (Fig. 2, *a*) is located 3 km southwest of the southern breakwater of Sevastopol Bay and extends 2 km inland. To the west of Streletskaya Bay, Kruglaya Bay (Fig. 2, *b*), almost circular in shape, is found. This bay is shallow with a sandy bottom. Winding Kamyshovaya Bay (Fig. 2, *c*), about 2.5 km long, is separated by two protective breakwaters limiting its entrance. Kazachya and Solyonaya bays (Fig. 2, *d*) together form Dvoynaya Bay, which is one of the most complex bays in the Sevastopol bays system. It is located 15 km west of the Sevastopol city centre, between Kamyshovaya Bay and Cape Chersonese. Kazachya Bay is approximately 600 m longer than Solyonaya Bay.

Numerical hydrodynamic Advanced Circulation Model for Shelves, Coasts, and Estuaries (ADCIRC) was used for calculations [1), 2)]. A variant of the model [14] based on depth-averaged equations of motion was used in this case

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -g\frac{\partial \eta}{\partial x} - C_d\frac{U\sqrt{U^2+V^2}}{H} + A_h\frac{\Delta q_x}{H}, \quad (1)$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} = -g\frac{\partial \eta}{\partial y} - C_d\frac{V\sqrt{U^2+V^2}}{H} + A_h\frac{\Delta q_y}{H}, \quad (2)$$

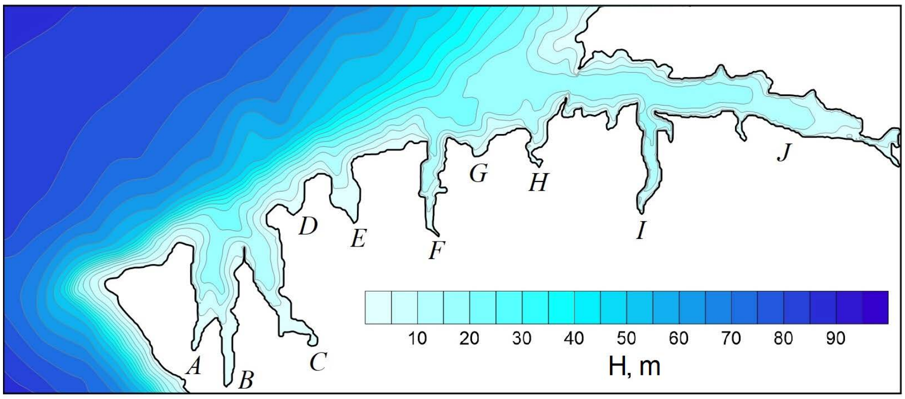
$$\frac{\partial \eta}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0\,. \quad (3)$$

Here, U, V are depth-averaged components of the velocity vector along axes x and y, respectively; η is basin water level; $H = h + \eta$ is dynamic depth; C_d is bottom friction coefficient; Δ is Laplace operator in spatial variables; A_h is horizontal turbulent viscosity coefficient; $q_x = UH$, $q_y = VH$ are components of the vector of total fluxes.

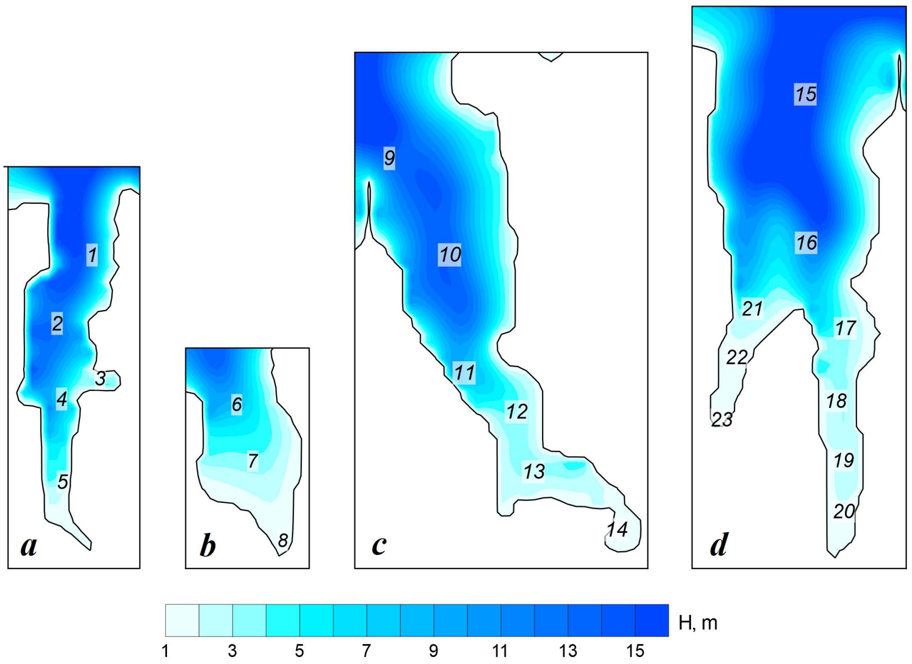
---

1) *Luettich R. A., Westerink J. J., Scheffner N. W.* ADCIRC: An Advanced Three-dimensional Circulation Model for Shelves Coasts and Estuaries. Report 1: Theory and Methodology of ADCIRC-2DDI and ADCIRC-3DL. Vicksburg, MS : U.S. Army Engineers Waterways Experiment Station, 1992. 137 p. (Dredging Research Program Technical Report DRP-92-6).

2) *Luettich R. A., Westerink J. J.* Formulation and Numerical Implementation of the 2D/3D ADCIRC. 2004. URL: https://adcirc.org/wp-content/uploads/sites/2255/2018/11/adcirc_theory_2004_12_08.pdf (дата обращения: 2.05.2025).



F i g . 1 . Bathymetry of the Sevastopol coastal zone. Notations: *A* – Solyonaya Bay, *B* – Kazachya Bay (these two bays constitute Dvoynaya Bay), *C* – Kamyshovaya Bay, *D* – Abramova Bay, *E* – Kruglaya Bay, *F* – Streletskaya Bay, *G* – Pesochnaya Bay, *H* – Karantinnaya Bay, *I* – Yuzhnaya Bay, *J* – Sevastopol Bay



F i g . 2 . Bathymetry of Sevastopol bays: *a* – Streletskaya; *b* – Kruglaya; *c* – Kamyshovaya; *d* – Dvoynaya. Numbers *1*–*23* stand for virtual mareographs (stations)

The numerical algorithm of the ADCIRC model is based on the finite element method using triangular elements and linear basis functions. To reduce computational noise during numerical integration of the system, the continuity equation is represented as so-called Generalized Wave Continuity Equation (GWCE)

$$\frac{\partial G}{\partial t} + \tau_0 G = 0,$$

where $G \equiv \partial\eta / \partial t + \partial q_x / \partial x + \partial q_y / \partial y$; τ_0 is non-negative parameter affecting the phase characteristics and stability of the numerical algorithm. After some identical transformations, the GWCE takes the following form

$$\frac{\partial^2 \eta}{\partial^2 t} + \tau_0 \frac{\partial \eta}{\partial t} + \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} = 0,$$

where

$$J_x = -q_x \frac{\partial U}{\partial x} - q_y \frac{\partial U}{\partial y} - \frac{g}{2}\frac{\partial \eta^2}{\partial x} - C_d \frac{U\sqrt{U^2+V^2}}{H} + A_h \frac{\Delta q_x}{H} + \tau_0 q_x + U\frac{\partial \eta}{\partial t} - gH\frac{\partial \eta}{\partial x},$$

$$J_y = -q_x \frac{\partial V}{\partial x} - q_y \frac{\partial V}{\partial y} - \frac{g}{2}\frac{\partial \eta^2}{\partial y} - C_d \frac{U\sqrt{U^2+V^2}}{H} + A_h \frac{\Delta q_y}{H} + \tau_0 q_y + V\frac{\partial \eta}{\partial t} - gH\frac{\partial \eta}{\partial y}.$$

The GWCE-based numerical algorithm suppresses short-wavelength noise effectively when solving the system of equations, without the need for artificial viscosity or distortion of the long-wavelength portion of the spectrum.

The quasi-linear version of the ADCIRC model was used, taking into account bottom friction, because it was assumed that the contribution of nonlinear terms in equations (1)–(3) could be neglected. The Coriolis force was not taken into account due to its weak influence on the scale of Sevastopol bays. Horizontal turbulent viscosity was also not taken into account, so turbulent viscosity coefficient A_h was assumed to be equal to zero; parameter τ_0 was 0.005. Bottom friction coefficient $C_d = C_0\left[1+\left(H_b / H\right)^{\alpha}\right]^{\beta/\alpha}$, where C_0 is minimum value C_d; $\alpha = 10$ is dimensionless parameter that determines the rate of bottom friction growth when the depth is greater or less than the depth of wave breaking $H_b = 1$ m; $\beta = 1/3$ is dimensionless parameter that determines how bottom friction increases with decreasing basin depth.

Numerical modelling was carried out on unstructured computational grids, which numbered from ~4,000 to ~12,200 finite elements for different bays. The grid step ranged from 20 to 100 m. The time integration step was $\Delta t = 0.025$ s.

Waves in the bays water areas were excited by setting red-noise disturbance [15] at the liquid boundary of each bay under consideration.

The boundary condition on the liquid boundary has the following form

$$\zeta_b = \begin{cases} \zeta_p, & t \le t_p \\ 0, & t > t_p \end{cases}, \quad (4)$$

where $\zeta_p(x,y,t)$ is a random function with red-noise spectrum; t_p is pumping time.

The numerical experiment consisted of a 6 h pumping phase and a 6 h free oscillations phase. During the pumping stage, wave disturbances were generated at the liquid boundary of the computational domain due to condition (4). Using disturbances in the form of red-noise suppressed the high-frequency components and enabled the response of the bay system to be considered in the long-wave spectrum. During the free oscillations stage, the initial conditions were formed based on the results of calculations carried out in the previous stage. A free passage condition was set on the liquid boundary of the computational domain. Spectral analysis of the series of level deviations calculated in the free oscillation mode was used to identify the periods of modes with the highest intensity.

Modelling results and discussion

Numerical experiments were conducted to calculate the level oscillations at stations 1–23, which are located within the design area of each of the studied bays. Fig. 3 shows the calculated marigrams for the regime of free level oscillations.

The most intense seiche oscillations occur in Kamyshovaya (Fig. 3, *c*), Kazachya (Fig. 3, *d*) and Solyonaya (Fig. 3, *e*) bays. Additionally, damping of seiches in these bays is slower (within 2 h) than in Streletskaya and Kruglaya bays (Fig. 3, *a*, *b*) where water oscillations damping lasts ~45 min. It should be noted that Solenaya and Kazachya bays are part of Dvoynaya Bay (see Fig. 2, *d*), which results in higher intensity of seiches in more extended Kazachya Bay compared to Solyonaya Bay. This is consistent with the findings reported in [16], where a branched bay model with the characteristic dimensions, mean depth and configuration of Dvoynaya Bay was examined. The paper demonstrates that asymmetry results in a reduction in the intensity of oscillations in the relatively short bay, as well as in their significant amplitudes in the longer bay, and an expansion of the mode composition of seiche oscillations in both bays.

To determine the periods of seiche oscillation in the bays, the calculated marigrams were subjected to spectral analysis using scripts developed by Gert Klopman and *Delft Hydraulics* [3)]. The energy spectra of the oscillations of sea level $E(f)$ were obtained using the Fourier transform. The *ESD* spectral energy density in range of oscillations $[f_a; f_b]$ was calculated as

$$ESD = \int_{f_a}^{f_b} \left| E(f) \right|^2 df \; . \quad (5)$$

[3)] *Winde H. P.* Wave height from pressure measurements. 2012. 49 p. Available at: https://repository.tudelft.nl/record/uuid:e3b07efd-1ce9-4fd1-b051-c794c72959ca [Accessed: 12 December 2024].

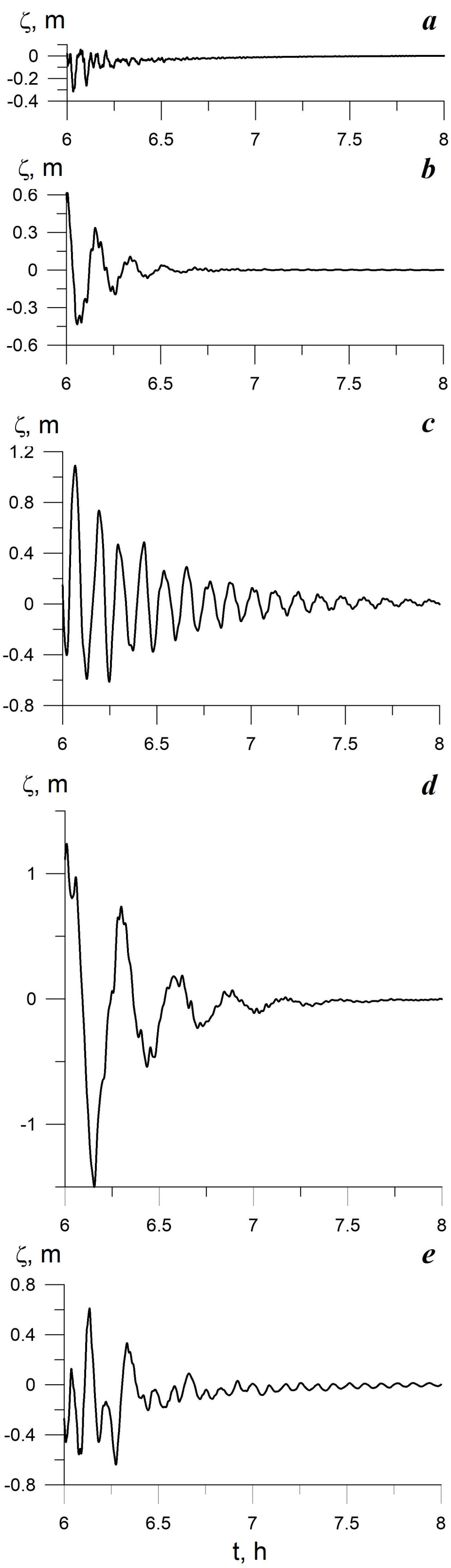
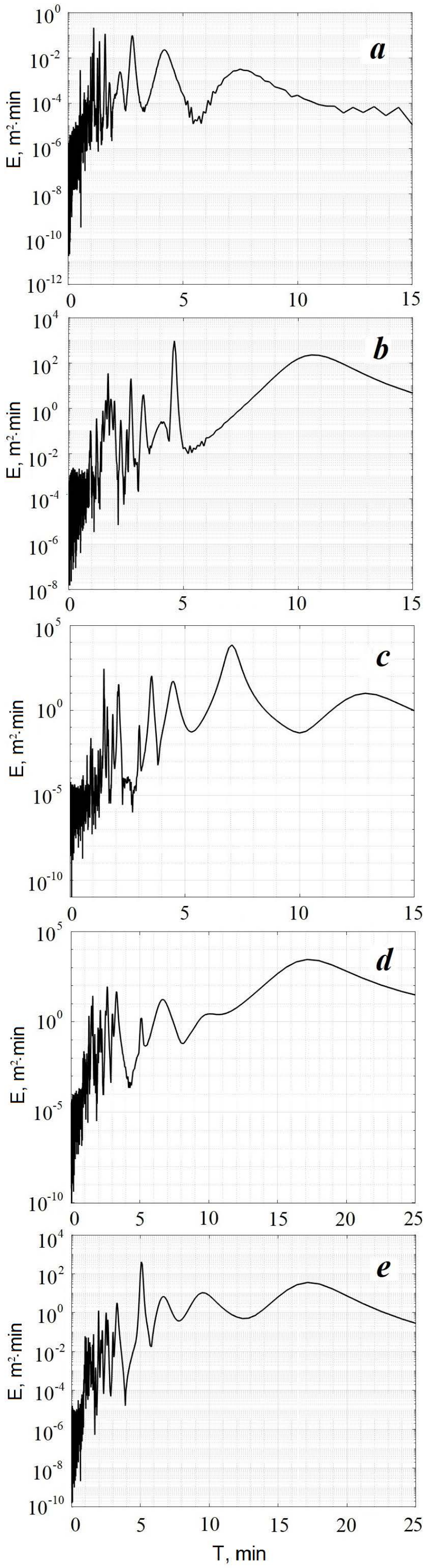


Fig. 4 shows the energy spectra of the oscillations of sea level $E(f)$ in the tops of the Sevastopol bays. It can be seen that modes with periods 1.1, 1.6, 2.8, 1.4 min are dominant in Streletskaya Bay (Fig. 4, *a*). In Kruglaya Bay, the main peaks are noted at periods 4.6, 10.6, 1.7, 2.7 min (Fig. 4, *b*). Modes with periods 7.1, 1.5, 3.6, 4.5, 2.1 min prevail in Kamyshovaya Bay (Fig. 4, *c*). In fact, Kazachya and Solyonaya bays are interconnected, as a result of which the mode composition of their seiches expands due to their mutual influence. The main peaks occur at periods 17.1, 2.7, 3.3, 1.6, 6.7 min in Kazachya Bay and at periods 5.1, 17.1, 9.5, 6.7, 3.3, 2.0, 2.5 min in Solyonaya Bay. Therefore, three similar peaks at periods 17.1, 3.3, 6.7 min can be seen in both bays.

Table shows the periods determined for the resonance oscillation modes of the bay tops, ordered by decreasing oscillation energy. Also given here are the periods of natural sea level oscillations in the Sevastopol bays calculated in [1] using formulas for a stationary-depth basin and a parabolic-bottom basin (these values are given in brackets) from [17]. As can be seen from Table, the values obtained in most periods in the present study coincide with the values of the periods determined analytically in [1] or close to them.

F i g . 3 . Marigrams calculated for the free oscillations mode in Sevastopol bays: *a* – Streletskaya (St. *5*); *b* – Kruglaya (St. *8*); *c* – Kamyshovaya (St. *14*); *d* – Kazachya (St. *20*); *e* – Solyonaya (St. *23*)

The absence of certain values and the emergence of new ones in our results are related to complex, non-rectangular shapes of the bays and their bathymetry peculiarities. Thus, in Streletskaya Bay, all of the previously identified periods were detected, except for the 13.5 min period corresponding to the Helmholtz mode of this bay. This appears to be connected with the impact of red-noise disturbance. In Kruglaya Bay, new modes with periods of 4.6, 1.9, 1.6 min appear. The present studies revealed a mode with a 12.9 min period in Kamyshovaya Bay while analytical estimates indicated a maximum period of 19.8 min. As for Kazachya and Solyonaya bays, which together form Dvoynaya Bay, almost all of the periods identified through analysis were detected in these bays. At the same time, the mode composition of sea level oscillations is significantly expanded due to the bays being connected to each other via entrances. The period values calculated for Kazachya (6.7, 2.5, 2.0 min) and Solyonaya (6.7, 3.3, 2.0, 2.5, 2.7 min) bays also agree well with the values obtained in [16] for the model bay, which has the same configuration and average depth as Dvoynaya Bay. At the same time, the present studies revealed a wider range of periods for these bays.



F i g . 4. Energy spectra of sea level oscillations resulting from red-noise disturbance in Sevastopol bays: *a* – Streletskaya (St. *5*); *b* – Kruglaya (St. *8*); *c* – Kamyshovaya (St. *14*); *d* – Kazachya (St. *20*); *e* – Solyonaya (St. *23*)

Periods of natural oscillations in Sevastopol bays, min, for a stationary-depth basin and a parabolic-bottom basin

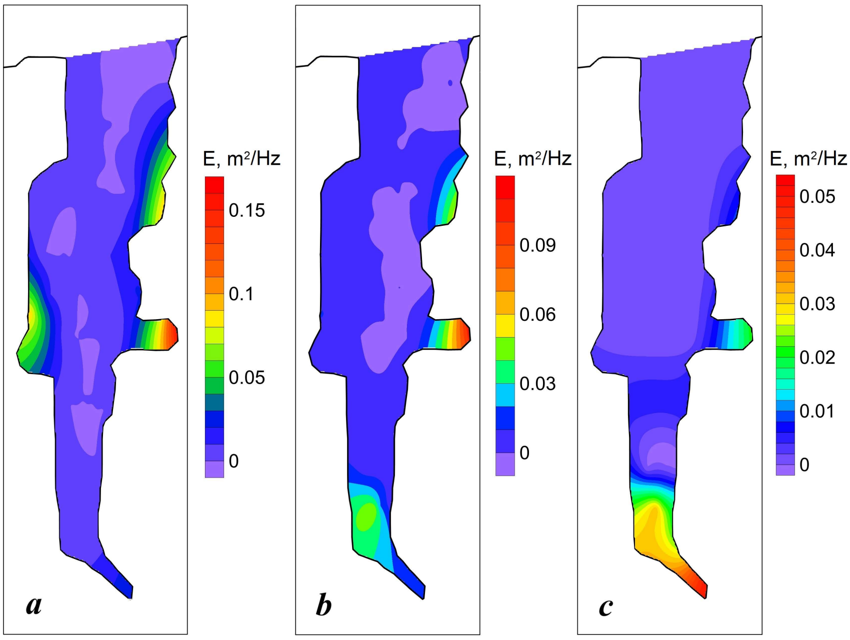
Bay (station)	Mathematically modelled periods, min	Periods from analytical estimation, min [1]
Streletskaya Bay (St. *5*)	**1.3** **1.6** **2.8** **1.4** **4.2**	13.5 4.5 (**4.3**) **2.7 (2.7)** **1.9** (2.0) **1.3** **1.2**
Kruglaya Bay (St. *8*)	4.6 **2.7** **10.6** **1.7** **3.3** 1.9 1.6 **2.0** **1.5**	**10.2** 3.4 (**2.7**) 2.0 (**1.7**) **1.5** (1.3) **3.0** **2.3**
Kamyshovaya Bay (St. *14*)	**7.1** **1.5** **3.6** 4.5 **2.1** 12.9	19.8 **6.7** (6.0) 4.0 (**3.8**) **2.8 (2.8)** **1.5** 1.4
Kazachya Bay (eastern arm of Dvoynaya Bay) (St. *20*)	**5.1** **17.1** 6.7 2.7 3.3 1.6 **1.5** 2.1 2.5 **3.0** 10.0 1.3 2.0 **2.2**	**15.2** **5.1** **3.0** **2.2** **1.2**

Continued

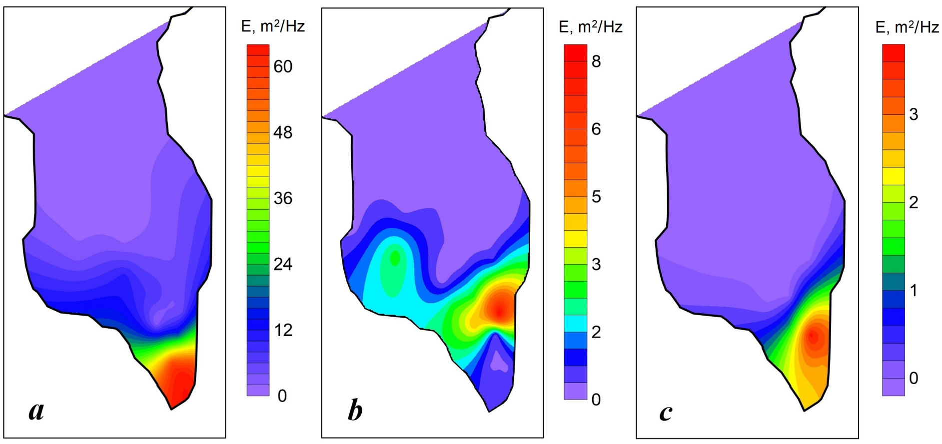
Bay (station)	Mathematically modelled periods, min	Periods from analytical estimation, min [1]
Solyonaya Bay (western arm of Dvoynaya Bay) (St. *23*)	5.1 17.1 **2.0** **9.5** 6.7 **3.3** 2.5 2.7 2.2 3.0 **1.6**	**9.6** **3.2** **1.9** **1.4** 1.3 1.2

N o t e . Values for the basin with parabolic bottom profile are given in brackets. Periods with good agreement are highlighted in bold. The periods in Kazachya and Solyonaya Bays, resulting from their interaction, are underlined.

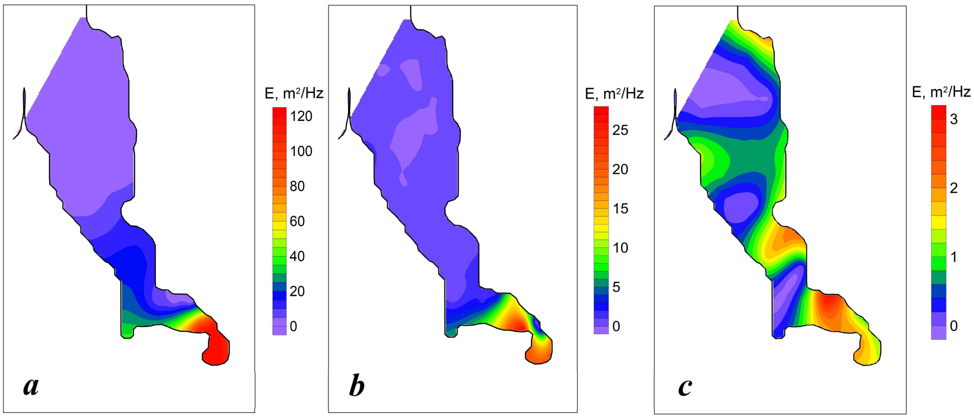
Figs. 5–8 demonstrate spatial distribution of the spectral energy density per unit time of the main energy-carrying level oscillations in Streletskaya, Kruglaya, Kamyshovaya, Dvoynaya bays calculated according to formula (5). For Streltskaya Bay (Fig. 5), the maximum values of the spectral energy density of seiche level oscillations are observed near the eastern coast of the bay and in its top (stations *3* and *5*). In Kruglaya Bay (Fig. 6), most of the energy of seiche oscillations is concentrated in its top (station *8*). Fig. 7 shows spatial distribution of spectral density of energy in Kamyshovaya Bay, from where we can see that the most intense oscillations are characteristic of the narrow part of the bay near its top (stations *12–14*). The most complex energy picture was obtained in the case of Dvoynaya Bay (Fig. 8). It can be seen that the resonance properties of the bay, which consists of two arms, are manifested in the intensification of level oscillations in both eastern arm (Kazachya Bay) and western arm (Solyonaya Bay). According to analytical estimates (Table), modes 5.1, 17.1 and 2.7 min are natural periods of Kazachya Bay (eastern arm of Dvoynaya Bay), therefore, the maximum values of the spectral energy density for these periods are noted in this bay (Fig. 8, *a*, *b*, *d*). According to the calculations, the mode with a 6.7 min period was also noticeably pronounced in Kazachya Bay (Fig. 8, *c*). The highest values of spectral energy



F i g . 5 . Spatial distribution of spectral density of main energy-carrying level oscillations in Streletskaya Bay for a period T of 1.3 min (*a*), 1.6 min (*b*), 2.8 min (*c*)



F i g . 6 . Spatial distribution of spectral density of main energy-carrying level oscillations in Kruglaya Bay for a period T of 4.6 min (*a*), 2.7 min (*b*), 10.6 min (*c*)



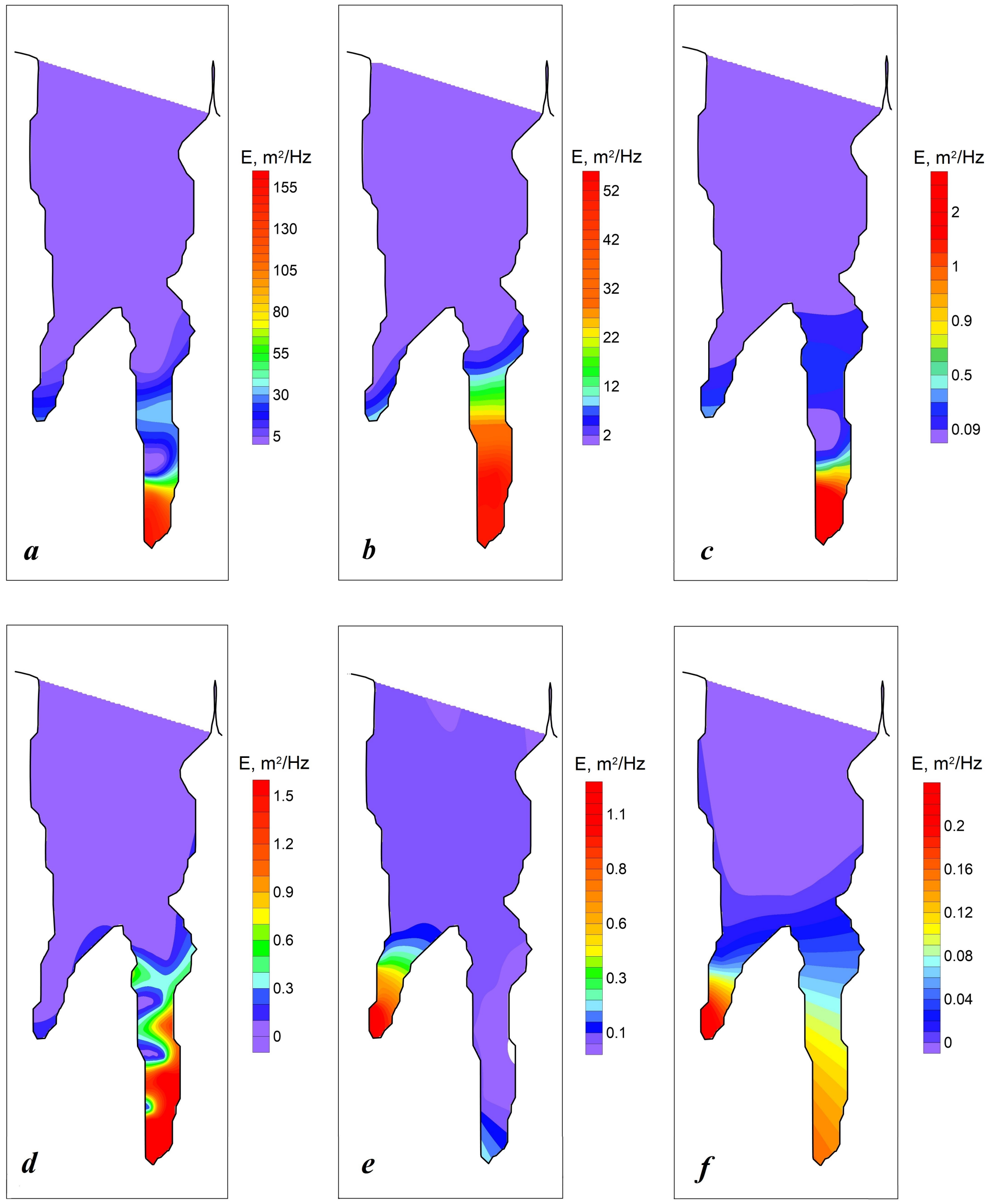
F i g . 7 . Spatial distribution of spectral density of main energy-carrying level oscillations in Kamyshovaya Bay for a period T of 7.1 min (*a*), 1.5 min (*b*), 3.6 min (*c*)

density were obtained for 2.0 and 9.5 min periods in Solyonaya Bay, as these are its natural periods (Solyonaya Bay is the western arm of Dvoynaya Bay) (Fig. 8, *e*, *f*).

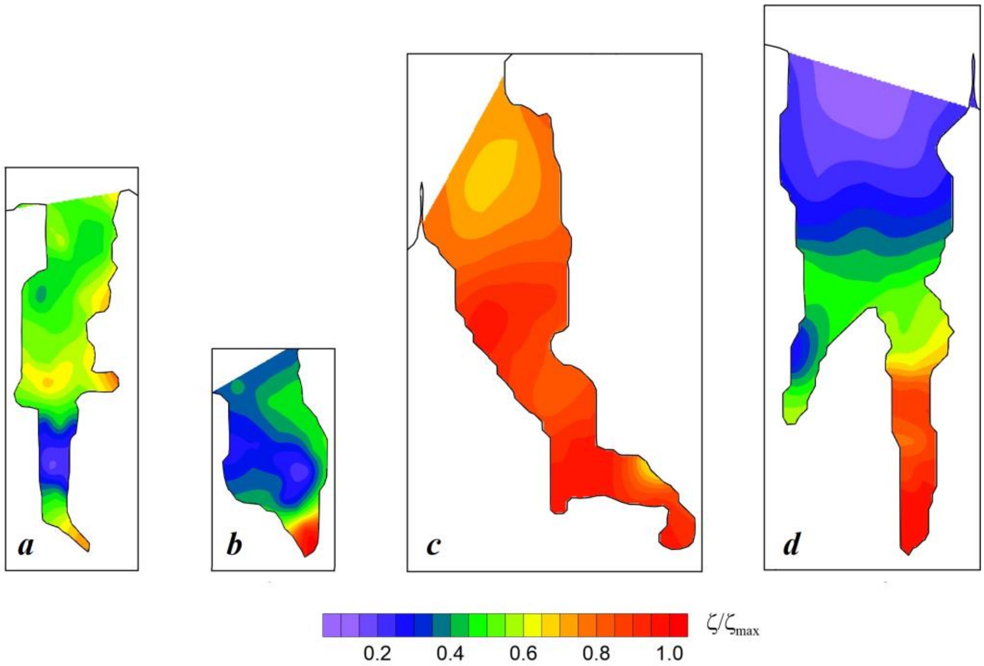
Analysis of the calculation results showed that Kazachya Bay (Fig. 8), with spectral density of over 150 m^2/Hz for a 5.1 min period, and Kamyshovaya Bay, with ~120 m^2/Hz for a 7.1 min period, had the highest values of spectral energy density of level oscillations. In Kruglaya Bay, maximum spectral energy density was ~65 m^2/Hz for a 4.6 min period. The lowest values of spectral energy density were observed in Streletskaya Bay (~0.15 m^2/Hz for a 1.3 min period).

Fig. 9 demonstrates the spatial distribution of relative (reduced to maximum values) amplitudes of sea level seiche oscillations in the bays. As can be seen, the largest amplitude values occur mainly in the tops of the bays, with the exception of Streletskaya Bay where the transverse mode dominates.

The spatial distribution of the amplitude and spectral density of level oscillations, as shown in Figs. 5–9, can be useful in identifying local zones where significant level rises caused by seiches are possible. This information is important for ensuring the safety of bay coastal infrastructure, such as ports.



F i g . 8 . Spatial distribution of spectral density of main energy-carrying level oscillations in Dvoynaya Bay for a period T of 5.1 min (*a*), 17.1 min (*b*), 6.7 min (*c*), 2.7 min (*d*), 2.0 min (*e*), 9.5 min (*f*)



F i g . 9 . Spatial distribution of relative amplitudes of level seiche oscillations in Sevastopol bays: *a* – Streletskaya; *b* – Kruglaya; *c* – Kamyshovaya; *d* – Dvoynaya

Conclusion

The hydrodynamic numerical model ADCIRC was used to study resonance properties of the water areas of the Sevastopol bays: Streletskaya, Kruglaya, Kamyshovaya, and Dvoynaya, which includes Kazachya and Solyonaya bays. Analytical estimates were used to control the resonance period values. The following conclusions can be drawn based on the analysis of the results of the calculations performed. The calculations were carried out for the water area of each bay separately. Waves in the bays water areas were excited by setting red-noise disturbance. The resonance periods of the above bays and spatial distribution of the spectral density of sea-level oscillations in their water areas were determined.

Most of the resonance periods identified for the Sevastopol bays are in satisfactory agreement with the analytical estimates of these periods. In Streletskaya Bay, however, it was not possible to excite the Helmholtz mode (the period of which, according to analytical estimates, is approximately 13.5 min) by setting red-noise disturbance. The period of the senior resonance mode of Kamyshovaya Bay was 12.9 min, which differs significantly from the analytical assessment of 19.8 min. This is because the bay water area has a complex spatial structure and two protective breakwaters built at its entrance.

The use of realistic bathymetry and shoreline profile data in the modelling allowed obtaining additional resonance periods for all the considered bays, which cannot be obtained with analytical estimates.

The spectral composition expansion of the resonance modes due to connection of these bays via their entrances was revealed in Dvoynaya Bay, which includes Kazachya and Solyonaya Bays.

The spectral density spatial distribution of the main energy-carrying sea level oscillations in Streletskaya, Kruglaya, Kamyshovaya and Dvoynaya Bays was analysed to show that the spectral density peaked mainly in the bay tops. In Dvoynaya Bay, the spectral density maxima manifested in the eastern or western arms (Kazachya or Solyonaya bays, respectively) depending on which of the arms their natural periods belonged to.

To clarify the resonance properties of the Sevastopol bays, *in situ* observations of the bay system are required.

The results of the study can be used in layout designing of hydraulic structures, development of mariculture, planning of wastewater outlets, etc.

REFERENCES

1. Manilyuk, Y.V., Lazorenko, D.I. and Fomin, V.V., 2021. Seiche Oscillations in the System of Sevastopol Bays. *Water Resources*, 48(5), pp. 726–736. https://doi.org/10.1134/S0097807821050122
2. Rabinovich, A.B., 1993. *Long Gravitational Waves in the Ocean: Capture, Resonance, and Radiation*. Saint Petersburg: Gidrometeoizdat, 326 p. (in Russian).
3. Alekseev, D.V., Manilyuk, Yu.V. and Sannikov, V.F., 2017. [Seiche Currents in an Open-Entry Basin]. In: S. O. Papkov, ed., 2017. *Applied Problems of Mathematics. Materials XXV of International Scientific and Technical Conference (Sebastopol, September 18–22, 2017)*. Sevastopol, pp. 109–115 (in Russian).
4. Dotsenko, S.F. and Ivanov, V.A., 2010. [*The Azov-Black Sea Region Nature Catastrophes*]. Sevastopol: ECOSI-Gidrofizika, 174 p. (in Russian).
5. Liu, P.L.-F., Monserrat, S., Marcos, M. and Rabinovich, A.B., 2003. Coupling Between Two Inlets: Observation and Modeling. *Journal of Geophysical Research: Oceans*, 108(C3), 3069. https://doi.org/10.1029/2002JC001478
6. Manilyuk, Yu.V., Lazorenko, D.I. and Fomin, V.V., 2020. Investigation of Seiche Oscillations in the Adjacent Bays by the Example of the Sevastopol and the Quarantine Bays. *Physical Oceanography*, 27(3), pp. 242–256. https://doi.org/10.22449/1573-160X-2020-3-242-256
7. Fomicheva, L.A., Rabinovich, A.B. and Demidov, A.N., 1991. [Sea Level]. In: E. N. Altman and A. I. Simonov, eds., 1991. *Hydrometeorology and Hydrochemistry of Seas in the USSR. Vol. IV. Black Sea. Issue 1. Hydrometeorological Conditions*. Leningrad: Gidrometeoizdat, pp. 329–354 (in Russian).
8. Morozov, A.N., Lemeshko, E.M., Shutov, S.A. and Zima, V.V., 2012. Currents in the Sevastopol bay from ADCP-observations (June, 2008). *Morskoy Gidrofizicheskiy Zhurnal*, (3), pp. 31–43 (in Russian).
9. Medvedev, I.P., 2018. Tides in the Black Sea: Observations and Numerical Modelling. *Pure and Applied Geophysics*, 175(6), pp. 1951–1969. https://doi.org/10.1007/s00024-018-1878-x
10. Goriachkin, Yu.N., Ivanov, V.A., Repetin, L.N. and Khmara, T.V., 2002. [Seiches in Sevastopol Bay]. In: UHMI, 2002. *Trudy UkrNIGMI = Proceedings of UHMI*. Kiev: UHMI. Vol. 250, pp. 342–353 (in Russian).

11. Manilyuk, Yu.V., Fomin, V.V., Yurovsky, Yu.Yu. and Bagaev, A.V., 2024. Sea Level Oscillations Spectra of a Shallow Coastal Bay: Cost-Effective Measurements and Numerical Modelling in Kruglaya Bay. *Regional Studies in Marine Science*, 69, 103326. https://doi.org/10.1016/j.rsma.2023.103326

12. Manilyuk, Yu.V., Lazorenko, D.I., Fomin, V.V. and Alekseev, D.V., 2023. Study of Seiche Oscillation Regimes in Sevastopol Bay. *Oceanology*, 63(6), pp. 796–805. https://doi.org/10.1134/s0001437023060115

13. Balinec, N.A. and Khmara, T.V., 2006. Phenomena of Harbor Seiches in the Sevastopol Bays. *Ecological Safety of Coastal and Shelf Zones and Comprehensive Use of Shelf Resources*, 14, pp. 204–208 (in Russian).

14. Fomin, V.V., Lazorenko, D.I. and Ivancha, E.V., 2017. Numerical Simulation of Seiches in the Balaklava Bay. *Ecological Safety of Coastal and Shelf Zones of Sea*, (3), pp. 32–39 (in Russian).

15. Rabinovich, A.B., Monserrat, S. and Fain, I.V., 1999. Numerical Modeling of Extreme Seiche Oscillations in the Region of the Balearic Islands. *Oceanology*, 39(1), pp. 12–19.

16. Belokon, A.Y. and Lazorenko, D.I., 2024. Energy Spectra of Sea Level Fluctuations During Propagation Long Waves in Branched Bays. In: V. Karev, ed., 2024. *Proceedings of the 9th International Conference on Physical and Mathematical Modelling of Earth and Environmental Processes 2023.* Springer Proceedings in Earth and Environmental Sciences. Cham: Springer, pp. 471–484. https://doi.org/10.1007/978-3-031-54589-4_49

17. Ivanov, V.A., Manilyuk, Yu.V., Sannikov, V.F., 2018. Seiches in a Basin with an Open Entrance. *Journal of Applied Mechanics and Technical Physics*, 59(4), pp. 594–600. https://doi.org/10.1134/S0021894418040041

Submitted 18.09.2024; accepted after review 23.12.2024;
revised 25.03.2025; published 30.06.2025

About the authors:

Yuri V. Manilyuk, Research Associate, Wave Theory Department, Marine Hydrophysical Institute of RAS (2 Kapitanskaya St., Sevastopol, 299011, Russian Federation), PhD (Phys.-Math.), **ORCID ID: 0000-0002-5752-7562, ResearcherID: P-6662-2017, Scopus Author ID: 6602563261**, *uvmsev@yandex.ru*

Aleksandra Yu. Belokon, Senior Research Associate, Department of Computational Technologies and Computational Modeling, Marine Hydrophysical Institute of RAS (2 Kapitanskaya St., Sevastopol, 299011, Russian Federation), PhD (Phys.-Math.), **ORCID ID: 0000-0002-1299-0983, ResearcherID: M-6839-2018**, *a.bazykina@mhi-ras.ru*

Andrei V. Bagaev, Leading Research Associate, Marine Hydrophysical Institute of RAS (2 Kapitanskaya St., 299011, Sevastopol, Russian Federation), PhD (Phys.-Math.), **ORCID ID: 0000-0003-4018-7642, ResearcherID: K-5373-2016**, *a.bagaev1984@mhi-ras.ru*

Yury Yu. Yurovsky, Leading Research Associate, Head of the Applied Marine Physics Laboratory, Marine Hydrophysical Institute of RAS (2 Kapitanskaya St., Sevastopol, 299011, Russian Federation), PhD (Phys.-Math.), **Scopus Author ID: 24377122700, ORCID ID: 0000-0002-9995-3965, ResearcherID: F-8907-2014**, *y.yurovsky@mhi-ras.ru*

Dmitry I. Lazorenko, Research Associate, Marine Hydrophysical Institute of RAS (2 Kapitanskaya St., Sevastopol, 299011, Russian Federation), PhD (Tech.), **ORCID ID: 0000-0001-7524-565X, ResearcherID: J-1925-2015**, *d.lazorenko.dntmm@gmail.com*

Contribution of the authors:

Yuri V. Manilyuk – statement of the problem, review of literature on the research problem, description of the research results, implementation of numerical experiments, analysis of the modeling results, writing the text of the article

Aleksandra Yu. Belokon – review of literature on the research problem, processing and description of the results of mathematical modeling, writing the text of the article, preparing graphic materials

Andrei V. Bagaev – processing and analysis of modeling results, preparation of the text of the article

Yury Yu. Yurovsky – processing and analysis of modeling results, preparation of the text of the article

Dmitry I. Lazorenko – preparation of input parameters for mathematical modeling, performing numerical experiments

All the authors have read and approved the final manuscript.